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A BRIEF INTRODUCTION TO FLUID MECHANICS

2ND EDITION

The weight, $\mathcal{W}$, of the air is equal to

$$\mathcal{W} = \rho g \times (\text{volume})$$
$$= (0.0102 \text{ slugs/ft}^3)(32.2 \text{ ft/s}^2)(0.84 \text{ ft}^3)$$

so that

$$\mathcal{W} = 0.276 \text{ lb} \qquad \textbf{(Ans)}$$

since 1 lb = 1 slug·ft/s^2.

1.6 Viscosity

V1.1 Viscous fluids

V1.2 No-slip condition

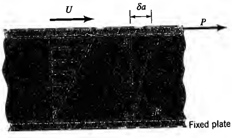
The properties of density and specific weight are measures of the "heaviness" of a fluid. It is clear, however, that these properties are not sufficient to uniquely characterize how fluids behave since two fluids (such as water and oil) can have approximately the same value of density but behave quite differently when flowing. There is apparently some additional property that is needed to describe the "fluidity" of the fluid.

To determine this additional property, consider a hypothetical experiment in which a material is placed between two very wide parallel plates as shown in Fig. 1.2. The bottom plate is rigidly fixed, but the upper plate is free to move.

When the force P is applied to the upper plate, it will move continuously with a velocity U (after the initial transient motion has died out) as illustrated in Fig. 1.2. This behavior is consistent with the definition of a fluid—that is, if a shearing stress is applied to a fluid it will deform continuously. A closer inspection of the fluid motion between the two plates would reveal that the fluid in contact with the upper plate moves with the plate velocity, U, and the fluid in contact with the bottom fixed plate has a zero velocity. The fluid between the two plates moves with velocity $u = u(y)$ that would be found to vary linearly, $u = Uy/b$, as illustrated in Fig. 1.2. Thus, a *velocity gradient, du/dy*, is developed in the fluid between the plates. In this particular case the velocity gradient is a constant since $du/dy = U/b$, but in more complex flow situations this would not be true. The experimental observation that the fluid "sticks" to the solid boundaries is a very important one in fluid mechanics and is usually referred to as the *no-slip condition*. All fluids, both liquids and gases, satisfy this condition.

In a small time increment, δt, an imaginary vertical line AB in the fluid (see Fig. 1.2) would rotate through an angle, $\delta\beta$, so that

$$\tan \delta\beta \approx \delta\beta = \frac{\delta a}{b}$$



■ **FIGURE 1.2 Behavior of a fluid placed between two parallel plates.**

Since $\delta a = U\,\delta t$ follows that

$$\delta\beta = \frac{U\,\delta t}{b}$$

Note that in this case, $\delta\beta$ is a function not only of the force P (which governs U) but al
of time. We consider the *rate* at which $\delta\beta$ is changing, and define the *rate of shearing stra*
$\dot{\gamma}$, as

$$\dot{\gamma} = \lim_{\delta t \to 0} \frac{\delta\beta}{\delta t}$$

which in this instance is equal to

$$\dot{\gamma} = \frac{U}{b} = \frac{du}{dy}$$

A continuation of this experiment would reveal that as the shearing stress, τ, is i
creased by increasing P (recall that $\tau = P/A$), the rate of shearing strain is increased in
rect proportion—that is

$$\tau \propto \dot{\gamma}$$

or

$$\tau \propto \frac{du}{dy}$$

This result indicates that for common fluids such as water, oil, gasoline, and air the sh
ing stress and rate of shearing strain (velocity gradient) can be related with a relationshi
the form

$$\tau = \mu \frac{du}{dy}$$

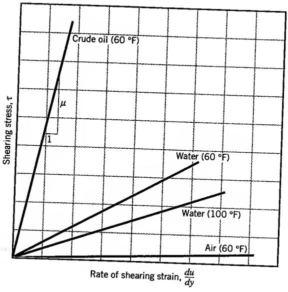
where the constant of proportionality is designated by the Greek symbol μ (mu) and is c
the *absolute viscosity, dynamic viscosity,* or simply the *viscosity* of the fluid. In accor
with Eq. 1.8, plots of τ versus du/dy should be linear with the slope equal to the vis
as illustrated in Fig. 1.3. The actual value of the viscosity depends on the particular f
and for a particular fluid the viscosity is also highly dependent on temperature as illus
in Fig. 1.3 with the two curves for water. Fluids for which the shearing stress is *lin*
related to the rate of shearing strain (also referred to as rate of angular deformatio
designated as *Newtonian fluids.* Fortunately most common fluids, both liquids and
are Newtonian. A more general formulation of Eq. 1.8 which applies to more complex f
of Newtonian fluids, is given in Section 6.8.1.

V1.3 Capillary tube viscometer

Fluids for which the shearing stress is not linearly related to the rate of shearing
are designated as *non-Newtonian fluids.* It is beyond the scope of this book to consi
behavior of such fluids, and we will only be concerned with Newtonian fluids.

From Eq. 1.8 it can be readily deduced that the dimensions of viscosity are FTL^{-2}
in BG units viscosity is given as $\text{lb}\cdot\text{s/ft}^2$ and in SI units as $\text{N}\cdot\text{s/m}^2$. Values of viscosity f
eral common liquids and gases are listed in Tables 1.4 through 1.7. A quick glance at t
bles reveals the wide variation in viscosity among fluids. Viscosity is only mildly dep
on pressure and the effect of pressure is usually neglected. However, as previously men
and as illustrated in Appendix B (Figs. B.1 and B.2), viscosity is very sensitive to tem

V1.4 Non-Newtonian behavior



■ FIGURE 1.3 **Linear variation of shearing stress with rate of shearing strain for common fluids.**

Quite often viscosity appears in fluid flow problems combined with the density in the form

$$\nu = \frac{\mu}{\rho}$$

This ratio is called the *kinematic viscosity* and is denoted with the Greek symbol ν (nu). The dimensions of kinematic viscosity are L^2/T, and the BG units are ft^2/s and SI units are m^2/s. Values of kinematic viscosity for some common liquids and gases are given in Table 1.4 through 1.7. More extensive tables giving both the dynamic and kinematic viscosities for water and air can be found in Appendix B (Tables B.1 through B.4), and graphs showing the variation in both dynamic and kinematic viscosity with temperature for a variety of fluids are also provided in Appendix B (Figs. B.1 and B.2).

Although in this text we are primarily using BG and SI units, dynamic viscosity is often expressed in the metric CGS (centimeter-gram-second) system with units of dyne·s/cm^2. This combination is called a *poise*, abbreviated P. In the CGS system, kinematic viscosity has units of cm^2/s, and this combination is called a *stoke*, abbreviated St.

EXAMPLE 1.3

A dimensionless combination of variables that is important in the study of viscous flow through pipes is called the *Reynolds number*, Re, defined as $\rho VD/\mu$ where ρ is the fluid density, V the mean fluid velocity, D the pipe diameter, and μ the fluid viscosity. A Newtonian fluid having a viscosity of 0.38 N·s/m^2 and a specific gravity of 0.91 flows through a 25-mm-diameter pipe with a velocity of 2.6 m/s. Determine the value of the Reynolds number using (a) SI units, and (b) BG units.